A CRASH COURSE IN CHARGED BLACK HOLES

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WHAT ARE BLACK HOLES?

SOLUTIONS TO EINSTEIN’S EQUATION

$$G^{\mu\nu} = 8\pi T^{\mu\nu}$$

curvature of spacetime **matter content**

WHAT KINDS OF BLACK HOLES ARE THERE?

TYPES OF BLACK HOLES

The No Hair Theorem tells us that black holes can be characterized by M, Q, and J

	$J = 0$	$J \neq 0$
$Q = 0$	Schwarzchild	Kerr
$Q \neq 0$	Reissner - Nordstrom	Kerr - Newman

WHY SHOULD WE STUDY R-N BH?

- simple solution (spherical symmetry!)
- similarities to Kerr (spinning BHs)
  - two horizons
  - spacetime diagrams
- connection to other areas of physics

WHAT EXACTLY ARE WE SOLVING FOR?

AN INTRODUCTION TO THE METRIC

Tells us the structure of spacetime and gives us an indication of how it curves.

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix}$$

- **The metric can be much more complicated!**
- **What is the metric for a Reissner-Nordstrom black hole?**

HOW DO WE FIND THE METRIC?

SPHERICAL SYMMETRY

Having spherical symmetry allows us to choose coordinates with a very simple metric!

$$g_{\mu\nu} = \begin{pmatrix} -e^{2\alpha(r)} & 0 & 0 & 0 \\ 0 & e^{2\beta(r)} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix}$$

THE EQUATIONS TO SOLVE

$$\boxed{G^{\mu\nu} = 8\pi T^{\mu\nu}}$$

Einstein's Field Equations

$$\nabla_\mu F^{\mu\nu} = J^\nu$$

Maxwell's Equations

EINSTEIN'S FIELD EQUATIONS

$$g_{\mu\nu} = \begin{pmatrix} -e^{2\alpha(r)} & 0 & 0 & 0 \\ 0 & e^{-2\alpha(r)} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix}$$

THE EQUATIONS TO SOLVE

$$G^{\mu\nu} = 8\pi T^{\mu\nu}$$

Einstein's Field Equations

$$\boxed{\nabla_\mu F^{\mu\nu} = J^\nu}$$

Maxwell's Equations

TOO MUCH ALGEBRA...

- $\nabla_0 F^{00} = \partial_0 F^{00} + \Gamma^0_{0\lambda} F^{\lambda 0} + \Gamma^0_{0\lambda} F^{0\lambda} = 0$
- $\nabla_1 F^{10} = \partial_1 F^{10} + \Gamma^0_{1\lambda} F^{\lambda 1} + \Gamma^1_{1\lambda} F^{0\lambda}$
  - $= -\dfrac{d}{dr} E(r) + \dfrac{1}{2} E(r)[g^{1\sigma}(2\partial_1 g_{1\sigma} - \partial_0 g_{11}) + g^{0\sigma}(2\partial_1 g_{1\sigma} - \partial_\sigma g_{11})]$
  - $= \cdots$
- **3 more equations!**

THE END RESULT

$$g_{\mu\nu} = \begin{pmatrix} -(1 - \frac{2M}{r} + \frac{Q^2}{r^2}) & 0 & 0 & 0 \\ 0 & (1 - \frac{2M}{r} + \frac{Q^2}{r^2})^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix}$$

WHAT DOES THE METRIC TELL US?

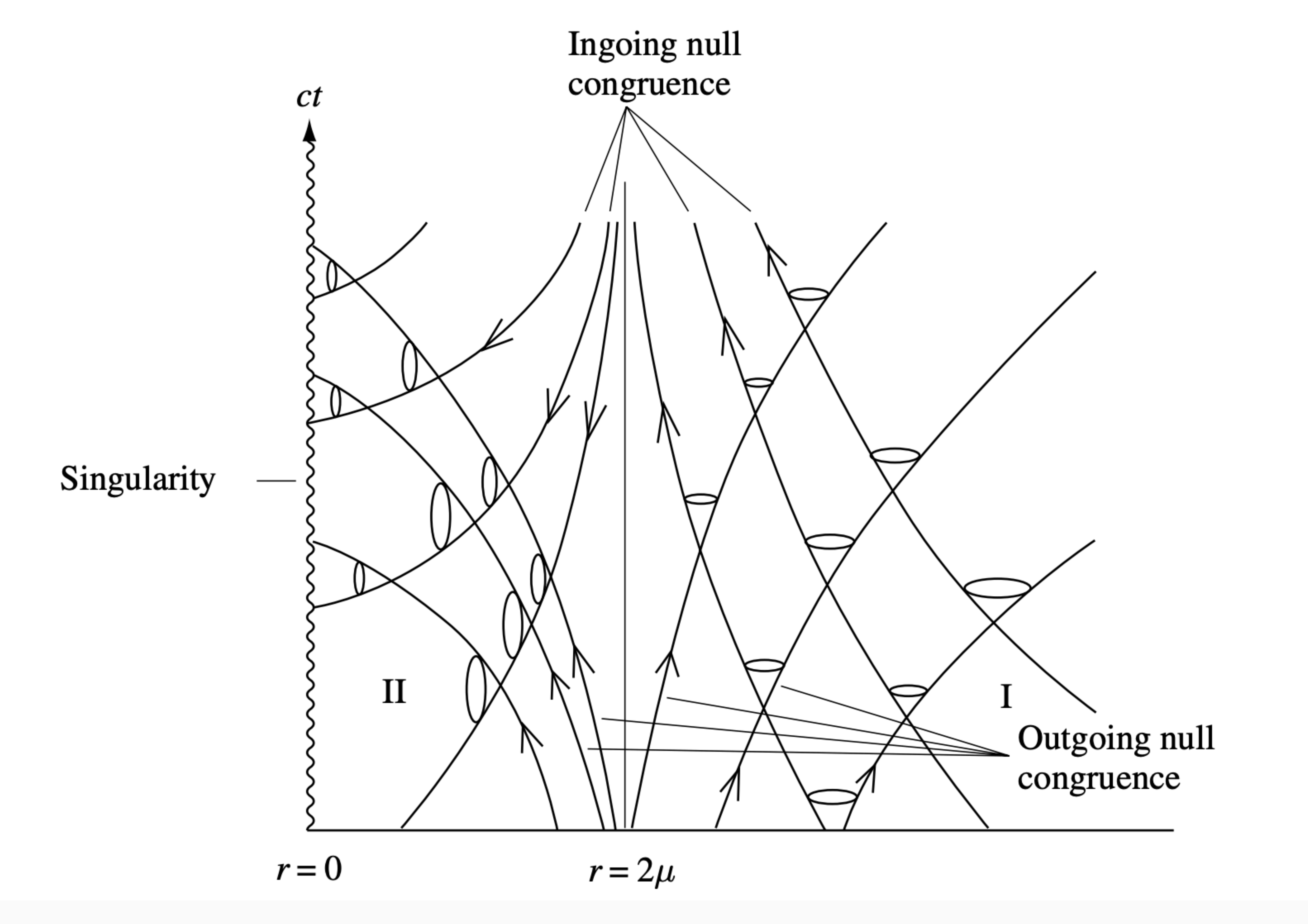
- at r =0 -> singularity
- at f = 0 -> not a singularity, but a horizon
  - two horizons!
  - $r_\pm = M \pm \sqrt{M^2 - Q^2}$
- $Q = 0 \implies r = 2M$

$$f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$$

$$g_{\mu\nu} = \begin{pmatrix} -(1 - \frac{2M}{r} + \frac{Q^2}{r^2}) & 0 & 0 & 0 \\ 0 & (1 - \frac{2M}{r} + \frac{Q^2}{r^2})^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix}$$

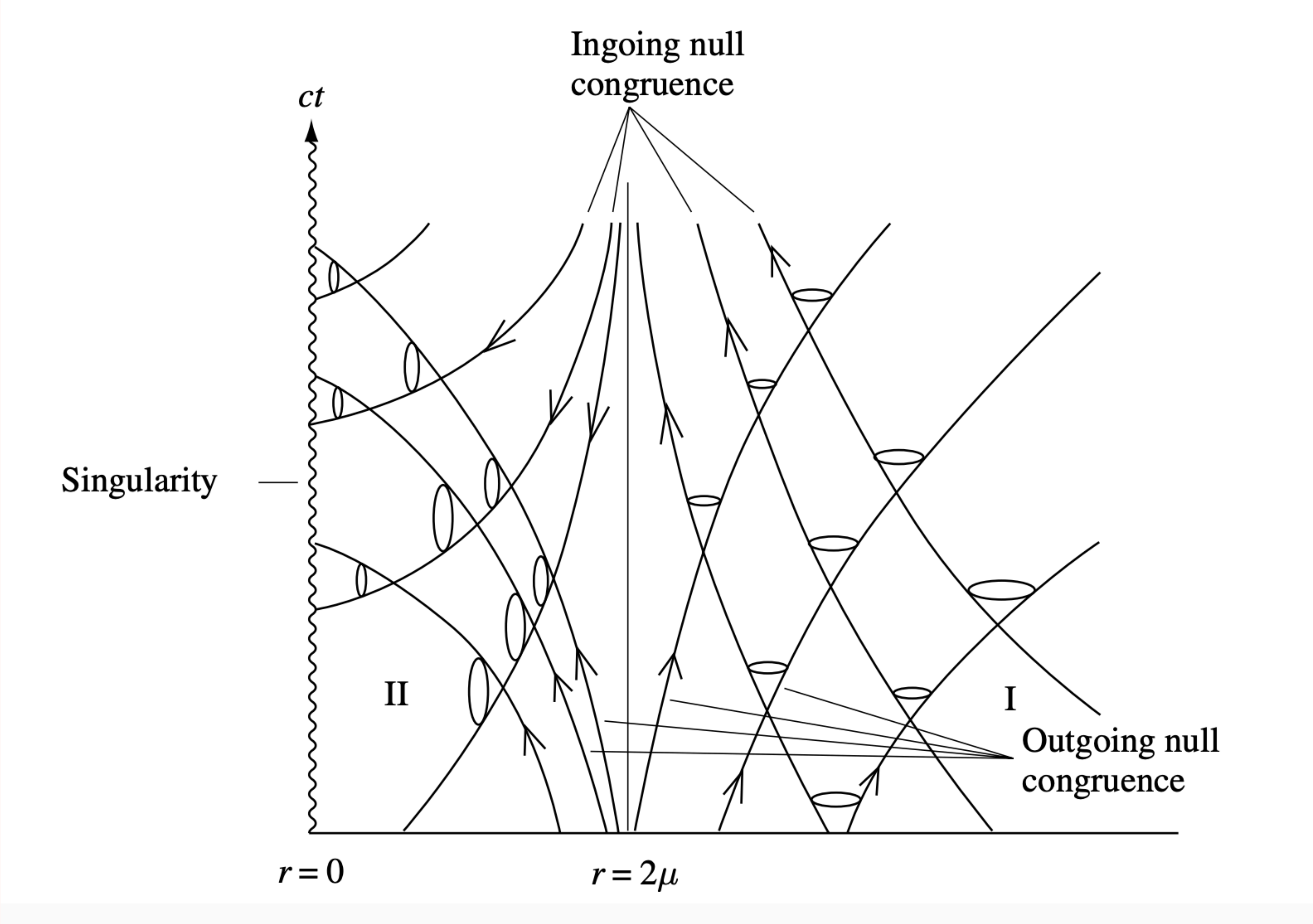
VISUALIZING BLACK HOLES

SPACETIME DIAGRAMS: SCHWARZSCHILD



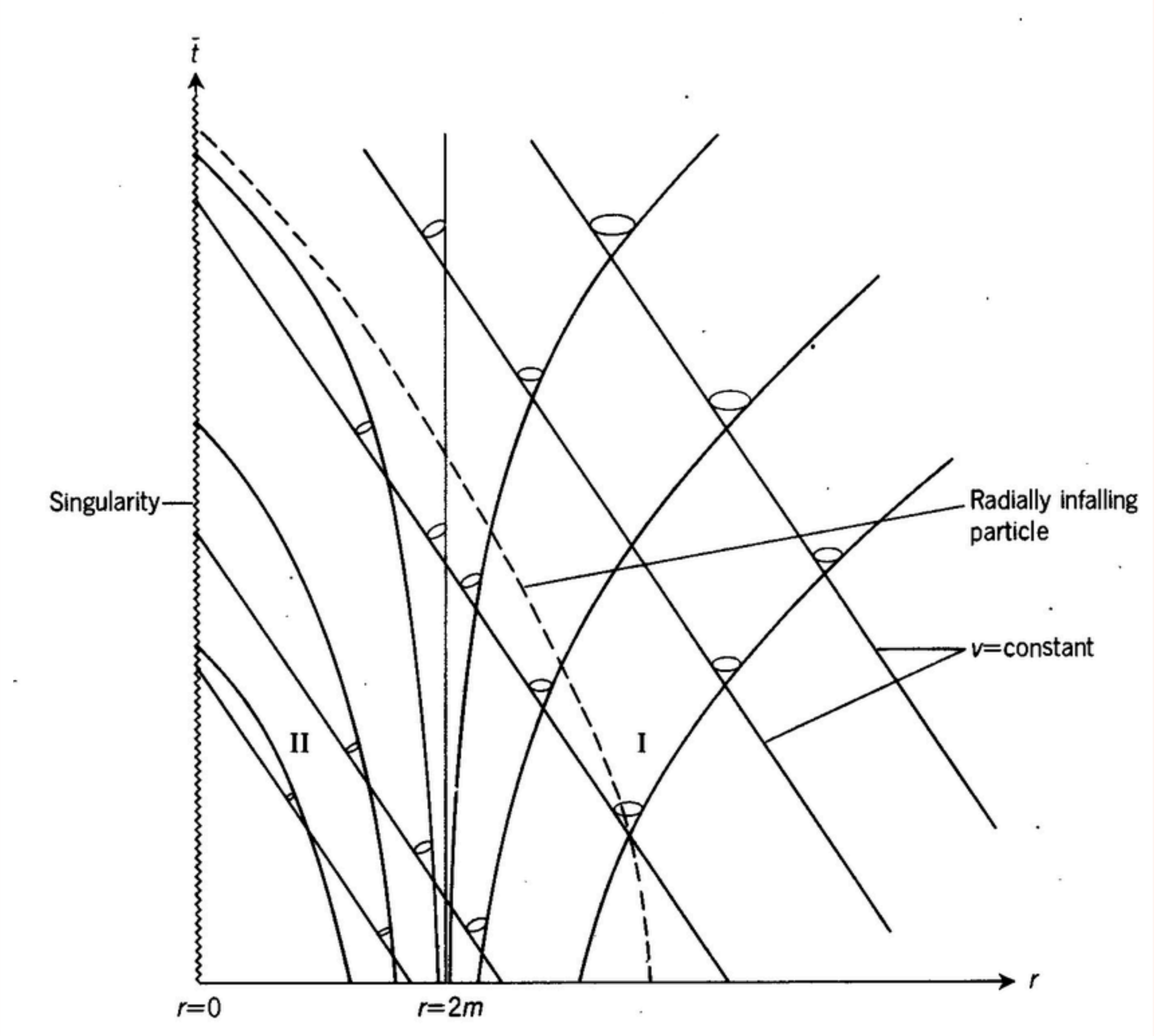
Schwarzschild Coordinates

SPACETIME DIAGRAMS: SCHWARZSCHILD



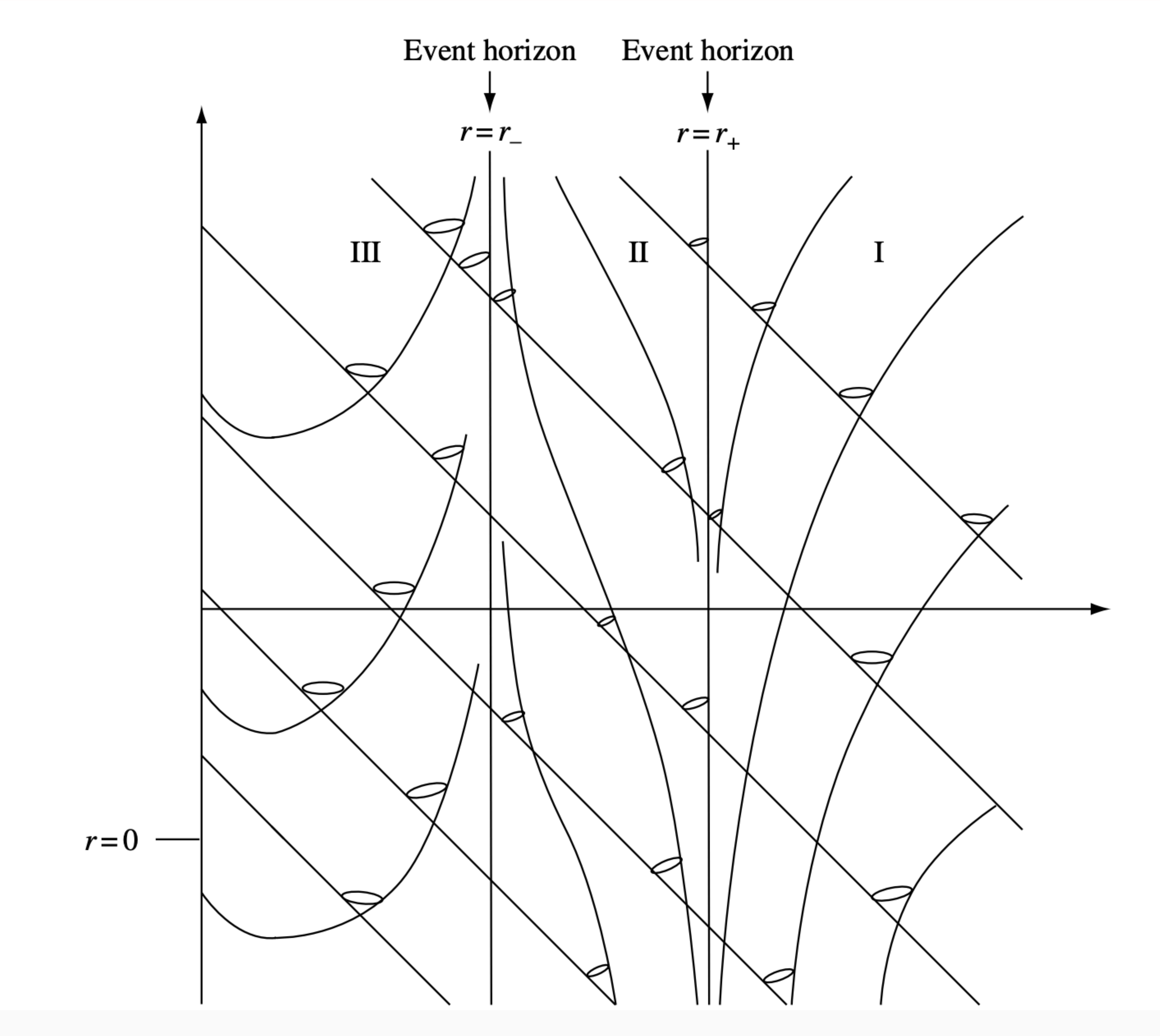
Schwarzschild Coordinates

$$\bar{t} = t + 2M\log(r - 2M)$$



Eddington Finkelstein Coordinates

SPACETIME DIAGRAMS: REISSNER-NORDSTROM



WHAT CAN WE LEARN FROM BLACK HOLES?

- Schwarzschild metric is a good approximation for large, spherically symmetric bodies
- Kerr (BH with spin)
  - astrophysics/gravitational waves
  - high energy physics
- AdS/CFT correspondence: general relativity to particle physics
  - Reissner-Nordstrom (BH with charge) -> high temperature superconductors

REFERENCES

- *Introducing Einstein's Relativity* by Ray d'Inverno
- *Einstein Gravity in a Nutshell* by Anthony Zee
- *Spacetime and Geometry* by Sean Carroll

THANK YOU FOR LISTENING!