" ...I drew an exponential through my noise. I believe the apparent legitimacy is enhanced by the fact that I used a complicated computer program to make the fit. I understand this is the same process by which the top quark was discovered." [1]

Using Statistics to Understand the Top Partner Off-Shell Mass

Connor Davis (student) Avik Roy (mentor)

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Background: Particle Physics and Feynman Diagrams

Subatomic interactions are governed by Feynman Diagrams

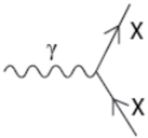
Figure 1: Feynman-mobile [2]

Using a few simple rules, we can describe and quantify most types of particle interactions...

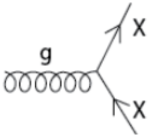
Background: Feynman Diagram Rules

- All diagrams are constructed from simple vertices
- Charge is always conserved
- Flavor is always conserved EXCEPT in weak interactions

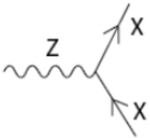
EM Vertex

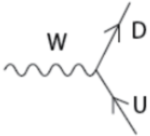


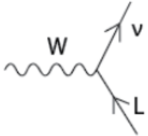
Strong Vertex



Weak Vertices







Background: On-Shell and Off-Shell

- Virtual particles are "off-shell"- they don't have to obey conservation of energy and momentum.

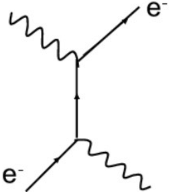
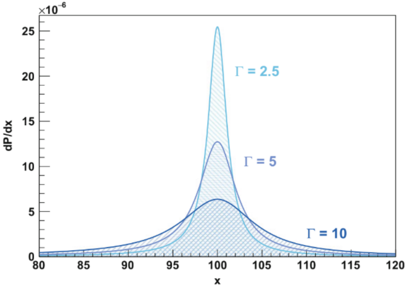


Figure 2: What process does this represent?

- "off-shell" masses are described by the relativistic Breit-Wigner Distribution [3].
$f(x) = \frac{N}{(x^2-m^2)^2+m^2\Gamma^2}$



The Top Partner Particle

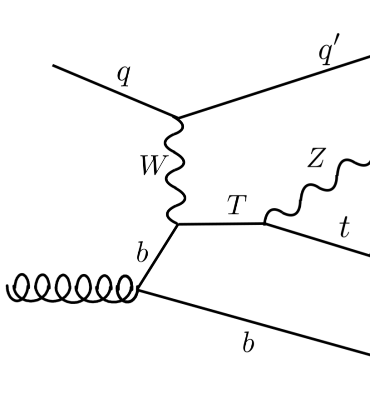


Figure 3: Process to produce a virtual top partner particle

- The top partner particle (T) is beyond the standard model.
- Simulated on-shell mass of 1500 GeV
- Proposed by many theories
- Interacts via:
  - T-W-b
  - T-Z-t
  - T-H-t

Simulation Process

- Used *Madgraph* to simulate 110,000 events.
- Each event consisted of:
  - The event is described by the energy and momentum of the associated particles
  - A calculation the top partner's off-shell mass
  - 20 tags, each containing a different Γ value and corresponding weight factor.
- *Madgraph* calculates the weights (from which we obtain the mass distribution) using matrix elements. The Breit-Wigner distribution uses the thin-width approximation.

Mass Histograms

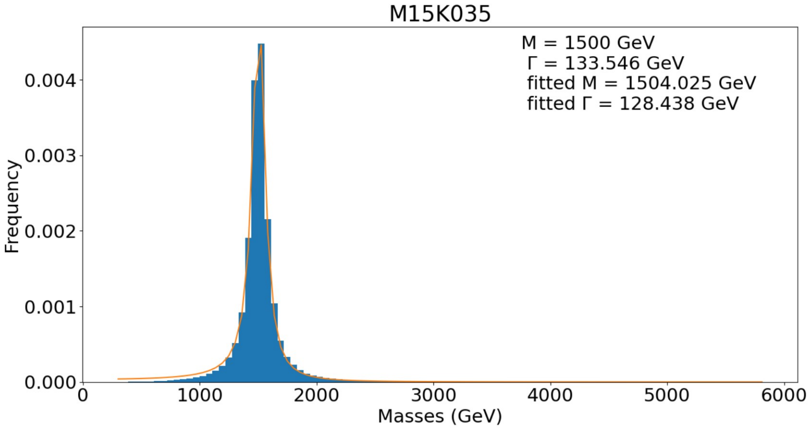


Figure 4: A great fit.

Mass Histograms

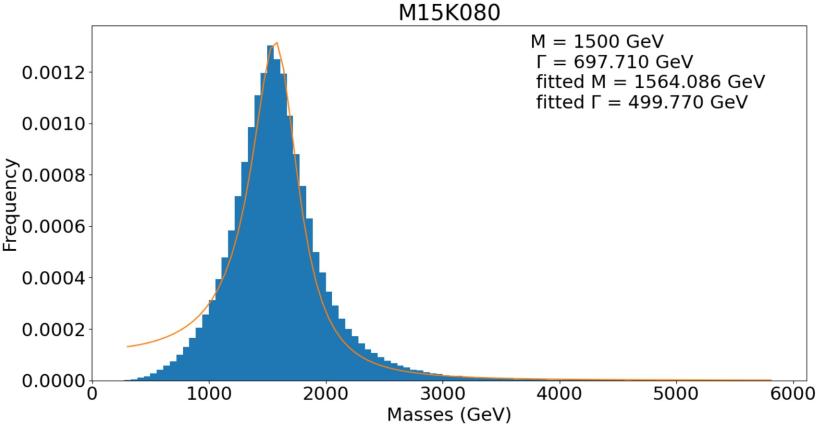


Figure 5: A decent fit.

Mass Histograms

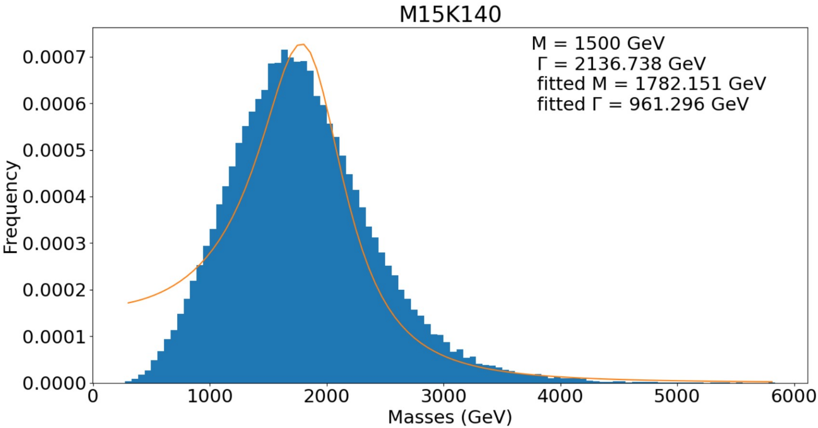


Figure 6: Good try, matplotlib. The thin-width approximation is no longer appropriate.

The Central Limit Theorem (CLT)

- Take a random sample of N points from an arbitrary distribution with mean μ standard deviation σ. The sampling distribution of the sample means will be normally distributed with a mean of μ and a standard deviation of $\sigma/\sqrt{N}$

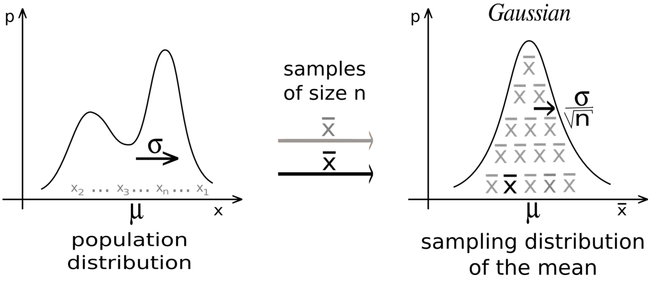


Figure 7: Visual Representation of the Central Limit Theorem [4]

The Central Limit Theorem Applied to the Relativistic Breit-Wigner Distribution

- The CLT is ubiquitous and incredibly useful. We hope to be able to use it on our data.
- But the relativistic Breit-Wigner distribution has infinite standard deviation.
  - Will the distribution still obey the central limit theorem?
  - If the CLT is obeyed, what is the relationship between Γ and σ?

Is the Sampling Distribution of the Sample Mean a Gaussian?

- Yes. For thin-width and wide-width distributions, the sampling distribution of the sample means is a Gaussian with finite standard deviation.
- The mean of the distribution shifts depending on the value of Γ.

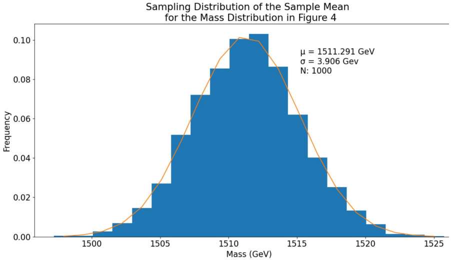


Figure 8: Sample mean distribution for a mass distribution with $\Gamma = 133.546$ GeV.

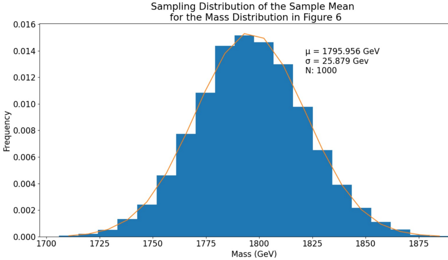
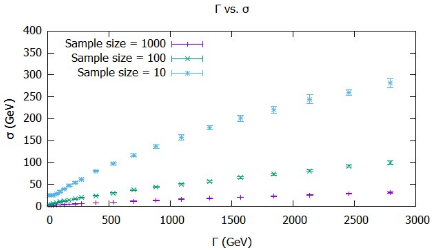
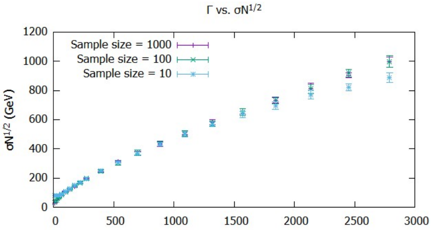


Figure 9: Sample mean distribution for a mass distribution with $\Gamma = 2136.738$ GeV.

Relating the sample size, σ, and Γ





- The Sapling distribution of the sample mean still obeys the $1/\sqrt{N}$ law.
- Γ is not directly proportional to σ.

Closing Remarks and Acknowledgements

- What is this project and what did I accomplish?
  - Exercise in data analysis and statistics
  - Improved programming skills
  - Increased the breadth and scope of my physics knowledge
- Thank you to Avik Roy and the directed reading program giving me the opportunity to pursue this project!

References

1. "Electron Band Structure in Germanium, ** ***", *Kovar/Hall*, http://pages.cs.wisc.edu/ kovar/hall.html.
2. "The Feynman Van", *Ralph Leighton*, http://www.feynman.com/fun/the-feynman-van/.
3. Luca, L. (2017). *Statistical Methods for Data Analysis in Particle Physics*. Springer.
4. Central limit theorem. Retrieved from https://en.wikipedia.org/wiki/Central_limit_theorem