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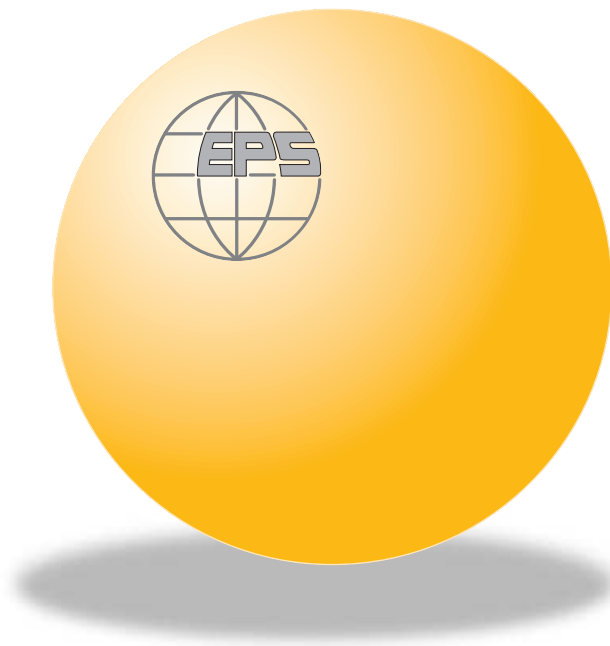
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Goldstone fluctuations in the amorphous solid state

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Abstract. – Goldstone modes in the amorphous solid state, resulting from the spontaneous breaking of translational symmetry due to random localisation of particles, are discussed. Starting from a microscopic model with quenched disorder, the broken symmetry is identified to be that of relative translations of the replicas. Goldstone excitations, corresponding to pure shear deformations, are constructed from long-wavelength distortions of the order parameter. The elastic free energy is computed, and it is shown that Goldstone fluctuations destroy localisation in two spatial dimensions, yielding a two-dimensional amorphous solid state characterised by power law correlations.

The *elastic* properties of gels and vulcanised matter have received considerable attention in the literature. Phenomenological arguments [1] as well as microscopic models [2] have been used to compute, *e.g.*, the shear modulus. The *structural* properties of gels have been discussed more recently [3], building on the work of Edwards and collaborators [4]. It has been shown that in the gel phase a finite fraction of particles are localised around random positions with finite thermal excursions. What has not been analysed so far, and what constitutes the main focus of this letter, is the connection between the spontaneously broken translational symmetry due to particle localisation and shear deformations, which are identified with Goldstone excitations as a consequence of the spontaneous breaking of translational symmetry.

We first discuss symmetry-related equilibrium states in the general context of amorphous solids. We construct low-energy long-wavelength Goldstone fluctuations, which correspond to almost uniform translations of the localised particles. Particular care is taken of the residual symmetry of the amorphous solid state, which remains statistically homogeneous. We then apply these general ideas to a particular type of amorphous solid —gels and vulcanised matter— for which a replica field theory is available [3]. This allows us to identify the lost symmetry to be that of relative translations of the replicas, whilst common translations of all replicas remain as intact symmetries in the amorphous solid state. Goldstone fluctuations are described as long-wavelength distortions of the order parameter, which are reminiscent of the ripples of an interface between coexisting liquid and gas phases. The free energy of these fluctuations is computed within a Gaussian approximation, and identified with the elastic free energy of a macroscopically homogeneous isotropic amorphous solid. A more comprehensive account of this work is given in ref. [5].

Symmetry breaking and Goldstone modes. – We consider a system of N distinguishable particles in a volume V interacting *via* potentials that are translationally invariant, *e.g.*, additive pair potentials. In the amorphous solid phase, a finite fraction of the particles are localised at random positions with thermal fluctuations of finite range. The system will be found in one of many possible equilibrium states. One such state is singled out arbitrarily, denoted by ν and specified by nonzero local static density fluctuations $\langle e^{i\mathbf{k}\cdot\mathbf{R}_j} \rangle_\nu$. Here, \mathbf{R}_j is the position of particle j and $\langle \dots \rangle_\nu$ denotes a thermal average restricted to equilibrium state ν . The amorphous state has macroscopic translational invariance, so that

$$\lim_{N \rightarrow \infty} \left[\frac{1}{N} \sum_{j=1}^N \langle e^{i\mathbf{k}\cdot\mathbf{R}_j} \rangle_\nu \right] = 0, \quad (1)$$

where $[\dots]$ denotes a disorder average. Localisation in the equilibrium state ν is detected by higher-order moments, the simplest one being the second:

$$\Omega_2(\mathbf{k}_1, \mathbf{k}_2) = \left[\frac{1}{N} \sum_{j=1}^N \langle e^{i\mathbf{k}_1\cdot\mathbf{R}_j} \rangle_\nu \langle e^{i\mathbf{k}_2\cdot\mathbf{R}_j} \rangle_\nu \right] = \delta_{\mathbf{k}_1+\mathbf{k}_2, \mathbf{0}} \omega(\mathbf{k}_1). \quad (2)$$

For this to be nonzero, macroscopic translational invariance requires $\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{0}$, and $\omega(\mathbf{k}_1)$ is a real-valued function. The second moment vanishes in the isotropic fluid phase and diagnoses the localisation of particles. Hence the second moment (and in fact all higher moments, as will be discussed below) serves as an order parameter for the transition from the fluid to the amorphous solid state.

In a different equilibrium state, denoted by μ , all particles are translated with respect to the equilibrium state ν by a displacement \mathbf{a}^μ , so that

$$\langle e^{i\mathbf{k}\cdot\mathbf{R}_j} \rangle_\mu = e^{i\mathbf{k}\cdot\mathbf{a}^\mu} \langle e^{i\mathbf{k}\cdot\mathbf{R}_j} \rangle_\nu. \quad (3)$$

Due to the translational invariance of the Hamiltonian there is in fact a manifold of symmetry-related states, generated by spatially uniform translations. How do we have to generalise the representation of the order parameter (2), in order to take into account the underlying translational symmetry of the Hamiltonian? For two equilibrium states, μ and ρ , the second moment is given by

$$\Omega_2^{\mu,\rho}(\mathbf{k}_1, \mathbf{k}_2) = \left[\frac{1}{N} \sum_{j=1}^N \langle e^{i\mathbf{k}_1\cdot\mathbf{R}_j} \rangle_\mu \langle e^{i\mathbf{k}_2\cdot\mathbf{R}_j} \rangle_\rho \right] = e^{i\mathbf{k}_1\cdot(\mathbf{a}^\mu - \mathbf{a}^\rho)} \delta_{\mathbf{k}_1+\mathbf{k}_2, \mathbf{0}} \omega(\mathbf{k}_1). \quad (4)$$

We introduce a longitudinal wave vector $\mathbf{k}_\parallel = (\mathbf{k}_1 + \mathbf{k}_2)/2$ and a transverse wave vector $\mathbf{k}_\perp = (\mathbf{k}_1 - \mathbf{k}_2)/2$ and rewrite the above expression

$$\Omega_2^{\mu,\rho}(\mathbf{k}_1, \mathbf{k}_2) = e^{i\mathbf{k}_\perp \cdot \mathbf{u}_\perp^{\mu,\rho}} \delta_{\mathbf{k}_\parallel, \mathbf{0}} \omega(\mathbf{k}_\perp) \quad (5)$$

in terms of a transverse displacement $\mathbf{u}_\perp^{\mu,\rho} = \mathbf{a}^\mu - \mathbf{a}^\rho$. Thereby, the second moment is represented by its ‘‘magnitude’’ $\omega(\mathbf{k}_\perp)$ and the generator of transverse translations $e^{i\mathbf{k}_\perp \cdot \mathbf{u}_\perp^{\mu,\rho}}$.

The interpretation of the order parameter is more intuitive for the second moment as a function of real-space variables: $V\widehat{\Omega}_2^{\mu,\rho}(\mathbf{x}_1, \mathbf{x}_2) = \widehat{\omega}(\mathbf{x}_1 - \mathbf{x}_2 - \mathbf{u}_\perp^{\mu,\rho})$. It does not depend on $\mathbf{x}_\parallel = (\mathbf{x}_1 + \mathbf{x}_2)/2$, and hence can be visualised as a rectilinear *hill*, whose position is specified by a pair of pure states. In a simple model of particle localisation in D dimensions, a fraction

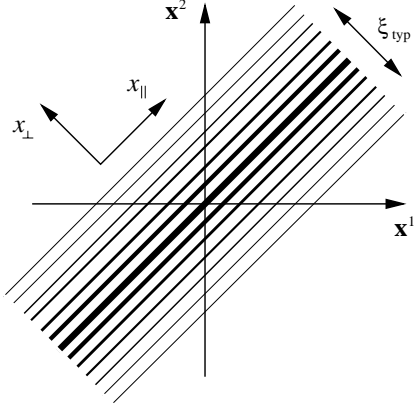


Fig. 1

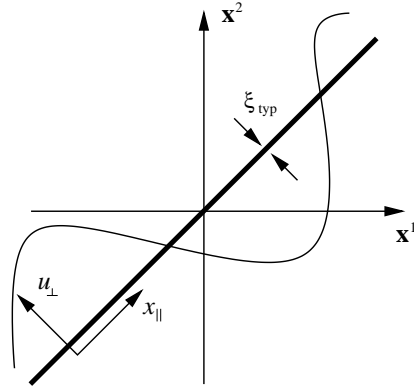


Fig. 2

Fig. 1 – Second moment in real space: a hill with its ridge aligned in the $\mathbf{x}_1 + \mathbf{x}_2$ direction. The thickness of the lines indicates the magnitude of the order parameter; the position of the ridge is determined by two equilibrium states.

Fig. 2 – Goldstone-distorted state in real space: the hill is displaced perpendicularly to the ridge by an amount u_\perp that varies with position x_\parallel along the ridge. Note that the scale of this figure is much larger than that used for fig. 1.

Q of the particles are harmonically bound to random positions with random mean-square displacements $D\xi_i^2$, giving rise to

$$\omega(\mathbf{k}_\perp) = Q \int_0^\infty d\xi^2 \mathcal{P}(\xi^2) e^{-\xi^2 k_\perp^2}. \quad (6)$$

Here $\mathcal{P}(\xi^2) \equiv [N_{\text{loc}}^{-1} \sum_{j \text{ loc}} \delta(\xi^2 - \xi_j^2)]$ is the disorder-averaged distribution of squared localisation lengths of the N_{loc} localised particles. The width of the hill is thus given by the typical value ξ_{typ} of the localisation length. The structure in one space dimension is sketched in fig. 1.

This pattern of symmetry breaking suggests that the Goldstone excitations of an equilibrium state are constructed from it *via* \mathbf{x}_\parallel -dependent translations of the hill in the $\mathbf{x}_\perp = (\mathbf{x}_1 - \mathbf{x}_2)/2$ direction, *i.e.*, *ripples* of the hill and its ridge (see fig. 2). In Fourier space, the generalisation of eq. (5) to inhomogeneous displacements reads

$$\Omega(\mathbf{k}_1, \mathbf{k}_2) = \frac{1}{V} \int d\mathbf{x}_\parallel e^{i\mathbf{k}_\parallel \cdot \mathbf{x}_\parallel + i\mathbf{k}_\perp \cdot \mathbf{u}_\perp(\mathbf{x}_\parallel)} \omega(\mathbf{k}_\perp). \quad (7)$$

The corresponding representation in real space is completely equivalent:

$$\widehat{\Omega}(\mathbf{x}_1, \mathbf{x}_2) = \frac{1}{V} \widehat{\omega}(\mathbf{x}_\perp - \mathbf{u}_\perp(\mathbf{x}_\parallel)). \quad (8)$$

Goldstone fluctuations in replica space. – In the following, we apply the general concepts of Goldstone excitations to a particular amorphous solid state —gels and vulcanised matter. For these systems a semi-microscopic model provides a framework for analysing the transition from the viscous fluid to the gel, as well as the structural and thermal properties of the gel, quantitatively. Here we use it to compute the free energy of Goldstone fluctuations, identify it with the elastic free energy, and thereby extract the shear modulus.

The transition from the viscous fluid to the gel is controlled by the number of crosslinks, connecting randomly chosen pairs of monomers. These monomers can be part of a larger prefabricated structure, such as chains consisting of M monomers or stars or dimers, etc. For the following, the internal structure of the molecules is almost irrelevant; it will only determine certain parameter values. The average over the quenched disorder (*i.e.* the number and location of the crosslinks) is performed with the help of the replica trick. The resulting, effectively uniform, theory is formulated in terms of a field $\widehat{\Omega}(x)$ defined over $(1+n)$ -fold replicated space x , so that the argument x means the collection of $(1+n)$ position D -vectors $\{\mathbf{x}^0, \mathbf{x}^1, \dots, \mathbf{x}^n\}$. According to the replica scheme of Deam and Edwards [4], the disorder-averaged free energy F is given by $-F/T = \lim_{n \rightarrow 0} (\mathcal{Z}_{1+n} - \mathcal{Z}_1)/n\mathcal{Z}_1$, where T is the temperature. The replica partition function \mathcal{Z}_{1+n} is given as a functional integral $\mathcal{Z}_{1+n} \sim \int \mathcal{D}\Omega e^{-\mathcal{S}_\Omega}$ over the field $\Omega(k)$, where $k \equiv \{\mathbf{k}^0, \mathbf{k}^1, \dots, \mathbf{k}^n\}$ denotes wave vectors conjugate to x .

The expectation value of the field is given by

$$\langle \Omega(k) \rangle = \left\langle N^{-1} \sum_{j=1}^N \prod_{\alpha=0}^n e^{i\mathbf{k}^\alpha \cdot \mathbf{R}_j^\alpha} \right\rangle_{\mathcal{S}_\Omega}. \quad (9)$$

For this to be nonzero, macroscopic translational invariance requires $\sum_{\alpha=0}^n \mathbf{k}_\alpha = \mathbf{0}$. The case with only one of the $(n+1)$ wave vectors nonzero corresponds to the (particle number) density (1). Density fluctuations are suppressed by excluded-volume interactions, which tend to maintain homogeneity. Therefore the theory, in its simplest version, contains only fluctuating fields for which at least two of the D -component entries in the argument $k \equiv \{\mathbf{k}^0, \mathbf{k}^1, \dots, \mathbf{k}^n\}$ are nonzero. We refer to such $D(n+1)$ -dimensional vectors as the higher-order replica sector (HRS). Fields with l nonvanishing D -dimensional wave vectors correspond to the l -th moment of the local density, so in particular $l=2$ yields the second moment discussed above. We consider a finite-dimensional system with a small number of nearest neighbours, implying that fluctuations of the local density are in general non-Gaussian and all moments need to be taken into account.

The *translational* invariance of the system is reflected in the invariance of the effective Hamiltonian \mathcal{S}_Ω under *independent translations of the replicas*. In the amorphous solid state this symmetry is broken down to invariance under the subgroup of common translations of all the replicas. The manifold of symmetry-related classical solutions (*i.e.* saddle points of \mathcal{S}_Ω) is given by

$$\Omega_{\text{cl}}(k) = e^{i\mathbf{k}_\perp \cdot \mathbf{u}_\perp} \delta_{\mathbf{k}_\parallel, \mathbf{0}} \omega(k_\perp), \quad (10)$$

in analogy to eq. (5). We have decomposed wave vectors into longitudinal and transverse components with the help of a complete orthonormal basis set in replica space $\{\epsilon^\alpha\}_{\alpha=0}^n$. The “longitudinal” direction in $(n+1)$ -dimensional replica space is the unit vector $\epsilon = \sum_{\alpha=0}^n \epsilon^\alpha / \sqrt{1+n}$, which is constant in replica space. All $D(n+1)$ -dimensional vectors can be expanded in this basis: *e.g.*, $k = \sum_{\alpha=0}^n \mathbf{k}^\alpha \epsilon^\alpha$ and decomposed into longitudinal (\parallel) and transverse (\perp) components:

$$\mathbf{k}_\parallel \equiv \sum_{\alpha=0}^n \mathbf{k}^\alpha / \sqrt{1+n} \quad \text{and} \quad k_\perp \equiv k - \mathbf{k}_\parallel \epsilon. \quad (11)$$

Analogous expressions hold for x and u . Invariance under common translations of all replicas is guaranteed in the classical solution (10) by the Kronecker delta, which enforces the longitudinal wave vector \mathbf{k}_\parallel to be zero. The manifold of symmetry-related states is generated by purely transverse displacements.

The order parameter (10) exhibits the structure in replicated x -space discussed above for the second moment of the local density in different equilibrium states. The classical state

$\widehat{\Omega}_{\text{cl}}(x)$ can be visualised as a rectilinear hill in x -space with contours of constant height oriented parallel to $\mathbf{x}_{\parallel} \epsilon$. Symmetry-related classical states are obtained by translating the hill rigidly, perpendicular to the ridge-line. Goldstone excitations from the classical state are constructed *via* \mathbf{x}_{\parallel} -dependent translations of the hill in the x_{\perp} -direction [6], *i.e.*, ripples of the hill and its ridge:

$$V\Omega(k) = \int d\mathbf{x}_{\parallel} e^{i\mathbf{k}_{\parallel} \cdot \mathbf{x}_{\parallel} + ik_{\perp} \cdot u_{\perp}(\mathbf{x}_{\parallel})} \omega(k_{\perp}), \quad (12a)$$

$$V\widehat{\Omega}(x) = \widehat{\omega}(x_{\perp} - u_{\perp}(\mathbf{x}_{\parallel})). \quad (12b)$$

We are interested here in the low-energy excitations only and, hence, consider Goldstone fluctuations, whose energy is expected to vanish with a power of the inverse wavelength of the ripples. We neglect all other fluctuations, which are expected to be massive. For consistency we thus should restrict ourselves to Goldstone fluctuations with wavelengths that are larger than a short-distance cutoff $l_{<}$. In particular, we require that the displacement $u_{\perp}(\mathbf{x}_{\parallel})$ varies on length scales larger than the width of the interface and take $l_{<} = \xi_{\text{typ}}$. The physical content of this restriction is that, to be meaningful, elastic deformations should occur on length scales longer than the typical localisation length of the particles in the elastic medium.

As discussed after eq. (9), the critical fields of the theory reside in the HRS. Hence we want to make sure that the distortions of the order parameter remain in this sector. This is guaranteed if the distortions fulfill the incompressibility constraint: $|\det(\delta_{a,b} + \partial_a u_b^{\alpha})| = 1$, which, for small-amplitude distortions, reduces to $\partial_a u_a^{\alpha} = 0$.

Elastic free energy and shear modulus. – For the present purposes the essential term in the effective Hamiltonian \mathcal{S}_{Ω} [7] is the “quadratic gradient” term,

$$\mathcal{S}_{\Omega} = \frac{1}{2} V c \sum_{k \in \text{HRS}} \xi_0^2 k^2 \Omega(k) \Omega(-k) + \mathcal{O}(k^4 \Omega^2, \Omega^3, k^2 \Omega^3), \quad (13)$$

where c and ξ_0 are, respectively, the number density and linear size of the underlying entities being constrained and HRS indicates that only wave vectors in the higher-order replica sector are to be included in the summation. Although eq. (13) was originally obtained *via* an expansion valid close to the critical point, such a quadratic gradient term is expected to feature in a description valid deep inside the solid state. If one is seeking—as we are—the dominant contribution to the elastic free energy governing the displacement field u_{\perp} (*i.e.* the leading-order term in u_{\perp} and \mathbf{k}_{\parallel}) then it is adequate to retain solely this quadratic gradient term.

Inserting the fluctuating field into \mathcal{S}_{Ω} and computing the *increase*, \mathcal{S}_u , due to the distortion u_{\perp} (*i.e.* the elastic free energy) gives the following contribution, which arises solely from the quadratic “gradient” term in \mathcal{S}_{Ω} :

$$\mathcal{S}_u = \frac{\rho_n}{2T} \int d\mathbf{x} \sum_{\alpha=0}^n (\partial_a u_b^{\alpha}(\mathbf{x})) (\partial_a u_b^{\alpha}(\mathbf{x})), \quad (14a)$$

$$\rho_n \equiv \frac{T c}{(1+n)^{1+\frac{D}{2}}} \int V^n d\mathbf{k}_{\perp} \frac{\xi_0^2 k_{\perp}^2}{nD} \omega^2(k_{\perp}). \quad (14b)$$

Here, summation over repeated Cartesian indices a, b is implied and the displacement vectors \mathbf{u}^{α} have to fulfill the constraint $\sum_{\alpha} \mathbf{u}^{\alpha}(\mathbf{x}) = \mathbf{0}$.

It was shown in refs. [7, 8] that in the classical state (10)

$$\omega(k_{\perp}) = Q \int_0^{\infty} d\xi^2 \mathcal{P}(\xi^2) e^{-\xi^2 k_{\perp}^2 / 2}, \quad (15)$$

with the fraction of localised particles $Q = 2a\tau/3g$ and the distribution of localisation lengths $\mathcal{P}(\xi^2) = (\xi_0^2/a\tau\xi^4) \pi(\xi_0^2/a\tau\xi^2)$, expressed in terms of a universal classical scaling function

$\pi(\theta)$. Here, $a\tau$ and g are, respectively, the control parameter for the density of constraints and the nonlinear coupling constant. A semi-microscopic model of vulcanised macromolecular matter yields $a = 1/2$, $\tau = (\mu^2 - \mu_c^2)/\mu_c^2$, $\xi_0^2 = M\ell_p^2/2D$ and $g = 1/6$, where μ controls the mean number of constraints (and has critical value $\mu_c = 1$), ℓ_p is the persistence length of the macromolecules, and M is the number of segments per macromolecule. If the network is built up from single monomers, corresponding to $M = 1$, then ℓ_p is interpreted as the length of the tether connecting two point-like particles.

We substitute this classical value of the order parameter into eq. (14b) and take the replica limit $n \rightarrow 0$ to obtain the shear modulus

$$\rho_0 = 2Tc\tau^3 \int d\theta d\theta' \frac{\pi(\theta)\pi(\theta')}{\theta^{-1} + \theta'^{-1}}. \quad (16)$$

The shear modulus increases continuously from zero with the third power of the distance from the gel point, in agreement with the previous microscopic calculation of ref. [2].

A simple scaling argument is also consistent: The free energy density scales with the correlation length ζ as $T\zeta^{-D}$. This is to be compared with the elastic free energy density of eq. (14a). Since $\partial_a u_b$ is dimensionless, this implies $\rho_0 \sim \zeta^{-D} \sim \tau^{D\nu}$. At $D = 6$ we find agreement, which is to be expected since our calculation is based on a Gaussian expansion around the classical solution.

Destruction of localisation in 2D and quasi-long-range order. – We discuss the effects of Goldstone fluctuations at the level of Gaussian fluctuations (14a) only. The elastic Green function is given by

$$\mathcal{G}_{ab}(\mathbf{r}) = \int_{2\pi/L}^{2\pi/l_<} d\mathbf{k} e^{-i\mathbf{k}\cdot\mathbf{r}} (\delta_{a,b} - \hat{\mathbf{k}}_a \hat{\mathbf{k}}_b) \frac{1}{k^2}, \quad (17)$$

where $\hat{\mathbf{k}} \equiv \mathbf{k}/|\mathbf{k}|$. Here, the integration over wave numbers is restricted to $1/L < k/2\pi < 1/l_<$, where L is the linear size of the system. The mean value of the distorted classical state,

$$\langle V\Omega(k) \rangle \approx \int d\mathbf{x}_\parallel e^{i\mathbf{k}_\parallel \cdot \mathbf{x}_\parallel} \left\langle e^{i\mathbf{k}_\perp \cdot \mathbf{u}_\perp(\mathbf{x}_\parallel)} \right\rangle_{\mathcal{S}_u} \omega(k_\perp) = \delta_{\mathbf{k}_\parallel, \mathbf{0}} \exp[-T\Gamma_D k_\perp^2/2\rho_0] \omega(k_\perp), \quad (18)$$

is determined by the local Green function $\mathcal{G}_{ab}(\mathbf{0}) = \delta_{a,b}\Gamma_D$. In space dimension $D \geq 3$, Γ_D is finite, so that the order parameter has a finite expectation value. The latter is reduced, due to Goldstone fluctuations, as one would expect: In addition to independent fluctuations of the localised particles there are also collective fluctuations —transverse phonons— which effectively increase the extent of spatial fluctuations of the localised particles. This is best seen by recalling the classical structure for ω , eq. (15):

$$\langle V\Omega(k) \rangle \approx \delta_{\mathbf{k}_\parallel, \mathbf{0}} Q \int d\xi^2 \mathcal{P}(\xi^2) e^{-k_\perp^2(\xi^2 + T\Gamma_D/\rho_0)/2}. \quad (19)$$

Order parameter correlations are given within the Gaussian approximation by

$$\langle V\Omega(\mathbf{x}, k_\perp) V\Omega^*(\mathbf{x}', k_\perp) \rangle \approx \omega^2(k_\perp) \exp\left[-\frac{T}{\rho_0}(\mathcal{G}_{ab}(\mathbf{0}) - \mathcal{G}_{ab}(\mathbf{x} - \mathbf{x}'))k_{\perp a} \cdot k_{\perp b}\right]. \quad (20)$$

In $D = 3$, the asymptotics of the elastic Green function for large separation $r = |\mathbf{x} - \mathbf{x}'|$ is well known, $\mathcal{G}_{ab}(\mathbf{r}) \approx (\delta_{a,b} + \hat{\mathbf{r}}_a \hat{\mathbf{r}}_b)/8\pi r$, implying for the correlator of the order parameter

$$\langle V\Omega(\mathbf{x}, k_\perp) V\Omega^*(\mathbf{x}', k_\perp) \rangle \approx \omega^2(k_\perp) \exp\left[-\frac{T\Gamma_D k_\perp^2}{\rho_0} + \frac{T(k_\perp^2 + (k_\perp \cdot \hat{\mathbf{r}})^2)}{8\pi\rho_0 r}\right], \quad (21)$$

where $\hat{\mathbf{r}} \equiv \mathbf{r}/r$.

In two space dimensions, Γ_D increases logarithmically with the system size, so that the expectation value of the order parameter (18) vanishes in the macroscopic limit. Goldstone fluctuations destroy localisation in two space dimensions, in agreement with the Hohenberg-Mermin-Wagner theorem. Nevertheless, a two-dimensional amorphous solid persists, which is characterised by algebraic decay of order parameter correlations, *i.e.*, quasi-long-range (random) order. The asymptotics of the elastic Green function for large spatial separation is given by $\mathcal{G}_{ab}(\mathbf{0}) - \mathcal{G}_{ab}(\mathbf{r}) \sim \frac{1}{4\pi} \delta_{a,b} \ln(r/l_<)$, implying an algebraic decay of the order parameter correlations with a nonuniversal exponent η_k :

$$\langle V\Omega(\mathbf{x}, k_\perp) V\Omega^*(\mathbf{x}', k_\perp) \rangle \sim (r/l_<)^{-\eta_k} \omega^2(k_\perp) \quad \text{with } \eta_k = \frac{\Gamma k_\perp^2}{4\pi\rho_0}. \quad (22)$$

This scenario is similar to the theory of two-dimensional regular solids, taking into account harmonic phonons only, as discussed in ref. [9]. The main difference is the absence of Bragg peaks at reciprocal lattice vectors. Instead, there is a divergence in the scattering function at $\mathbf{k}_\parallel = \mathbf{0}$, as can be seen by integrating the correlator, above, over all $\mathbf{r} = \mathbf{x} - \mathbf{x}'$ [9]: $S(\mathbf{0}, k_\perp) = \int d\mathbf{r} \langle V\Omega(\mathbf{r}, k_\perp) V\Omega^*(\mathbf{0}, k_\perp) \rangle \sim (L/l_<)^{2-\eta_k}$.

* * *

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