Inherent Stochasticity of Superconductor-Resistor Switching Behavior in Nanowires

Nayana Shah, David Pekker, and Paul M. Goldbart
Department of Physics, University of Illinois at Urbana-Champaign, 1110 West Green Street, Urbana, Illinois 61801, USA
(Received 16 November 2007; published 10 November 2008)

We study the stochastic dynamics of superconductive-resistive switching in hysteretic current-biased superconducting nanowires undergoing phase-slip fluctuations. We evaluate the mean switching time using the master-equation formalism, and hence obtain the distribution of switching currents. We find that as the temperature is reduced this distribution initially broadens; only at lower temperatures does it show the narrowing with cooling naively expected for phase slips that are thermally activated. We also find that although several phase-slip events are generally necessary to induce switching, there is an experimentally accessible regime of temperatures and currents for which just one single phase-slip event is sufficient to induce switching, via the local heating it causes.

DOI: 10.1103/PhysRevLett.101.207001 PACS numbers: 74.78.Na, 73.21.Hb, 74.25.Fy, 74.40.+k

Recent advances in the fabrication of ultranarrow superconducting wires—using carbon nanotube-[1] or DNA templating [2]—have spurred renewed interest in quasi-one-dimensional superconductivity. Phase-slip fluctuations, in which the phase of the order parameter between the left and right ends of the wire changes by a multiple of 2π, have long been known to be an important ingredient in determining the properties of superconducting wires [3,4]. It is well established that thermally activated phase slips (TAPS) can provide an intrinsic source of dissipation that results in wide superconducting transitions of widths of up to several Kelvins [1,3,5,6]. The latest generation of experiments on even narrower superconducting nanowires aims to investigate quantum phase slips (QPS), i.e., phase slips proceeding by quantum instead of thermal fluctuations. QPS are thought to be important in the quantum phase transitions from the superconducting to the normal state (depairing transition) [7–10] and from the superconducting to the insulating state (SIT) [11–13] that have been observed in various experiments [1,14–19]. These experiments have prompted new theoretical investigations of quantum phase transitions and the properties of QPS in general. Beyond their importance in phase transitions, understanding fluctuations is important for the use of nanowires as components in microelectronics, including as current-limiting switching elements [20] and qubits [21]. Although QPS are quite plausible, the experimental evidence for their observation has been mixed. In addition, there is no generally agreed-upon theory of these fluctuations and of the relevant phase transitions.

Typically, superconducting nanowires show hysteretic current-voltage characteristics, with switching from the superconducting to the resistive state being both abrupt and stochastic (i.e., the current at which switching occurs differs from run to run). Because of fluctuations, this switching occurs at a lower current than the one associated with the depairing phase transition. By studying experimentally the temperature dependence of the switching statistics, one can learn about the nature of phase-slip fluctuations, as one can with underdamped Josephson junctions [22]. This strategy works well at low temperatures, where the wires become hysteretic; thus it compliments traditional experiments, which study fluctuations using low-bias current measurements and become intrinsically difficult at low temperatures, where the wire resistance becomes very small. The analogy to Josephson junctions, however, breaks down because phase-slip dynamics in nanowires are strongly over-damped in most regimes, and thus a single phase slip does not, in general, result in a switching event. Instead, multiple consecutive phase slips are required to trigger switching.

Inspired by recent experiments [23], in this Letter we study the stochastic aspects of the superconducting-to-resistive switching dynamics of current carrying nanowires, an area that has not received much attention to date. We construct a stochastic model in the spirit of the steady state model of Ref. [24]. Our model consists of two ingredients: (i) stochastic phase-slips that heat the wire, and (ii) heat dissipation that cools the wire. As the phase-slip rate depends on the local temperature of the wire, heating by phase slips can create a runaway cascade that eventually overheats the wire. We derive an equation for the dependence of the mean switching time on current and temperature that describes this model, and solve it numerically.

Having in mind the configuration adopted in recent and ongoing experiments on superconducting nanowires, we consider a freestanding wire of effective length L and cross-sectional area A, the ends of which are held at a fixed temperature T_b, as shown in Fig. 1. L may differ slightly from the geometric length of the wire to compensate for the heat spread in the lead at the wire attachment point. We concentrate on wires in the dirty limit, for which the mean free path is much shorter than the coherence length, which is shorter than the charge imbalance length required for thermalization, which itself is shorter than L. As the wire is suspended, essentially all heat generated locally in the wire $[Q(x, t)]$ can be taken away only through the ends. The

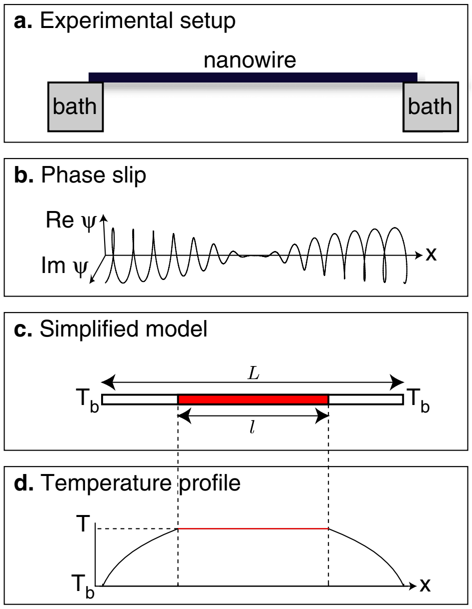

FIG. 1 (color online). (a) Schematic of the experimental configuration described by our model: a superconducting nanowire is suspended between two thermal baths. (b) Sketch showing the attenuation of the order parameter in the core of a phase-slip. (c) Schematic of the simplified model. All phase slips are taken to occur in a central (i.e. shaded) segment of length l, which is assumed to be at a uniform temperature T; heat is carried away through the end segments, which are assumed to have no heat capacity. The temperature at the ends of the wire is fixed to be T_b. (d) Sketch of a typical temperature profile.

corresponding heat conduction equation for the temperature $\Theta(x, t)$ at position x along the wire at time t reads

$$C_v(\Theta)\partial_t\Theta(x, t) = \partial_x[K_s(\Theta)\partial_x\Theta(x, t)] + Q(x, t), \quad (1)$$

and is characterized by the specific heat $C_v(\Theta)$ [25] and thermal conductivity $K_s(\Theta)$ [26] of the wire, together with the boundary condition $\Theta(\pm L/2, t) = T_b$ at its ends. Although our analysis rests on the premise that there are no additional heat-removing channels, it can readily be extended to account for such possibilities.

Our aim here is to study the stochasticity inherent in the switching process, and therefore it is necessary for us to explicitly take into consideration the fact that the resistive fluctuations of the superconducting nanowire consist of discrete phase-slip events (labeled by i) that take place at random instants and are centered at random spatial locations along the wire. To capture the essential physics, we shall consider the simpler model, represented in Figs. 1(c) and 1(d). Given that edge effects favor phase-slip locations away from the wire ends, the source term is restricted to the region near the center of the wire. The system is thus modeled by assuming that (i) heating takes place within a central segment of length l to which a uniform temperature T is ascribed, and (ii) the heat is conducted away through the end segments, within which we ignore the heat capacity. The length l can be roughly estimated to be of order the charge imbalance length required for thermalization. To simplify the problem further, we make use of the fact that the probability per unit time Γ_+ for an antiphase slip to take place is much smaller than the rate Γ_- for a phase slip to take place, and ignore the process of cooling by antiphase slips. To account indirectly for their presence, we use a reduced rate $\Gamma \equiv \Gamma_- - \Gamma_+$ instead of Γ_-. This ensures that the discrete expression for Q correctly reduces to continuous Joule heating.

With the model defined above, the description reduces to a stochastic ordinary differential equation for the time evolution of the temperature of the central segment:

$$\frac{dT}{dt} = -\alpha(T, T_b)(T - T_b) + \eta(T, I)\sum_i \delta(t - t_i). \quad (2)$$

The second term on the right-hand side corresponds to (stochastic) heating by phase slips that occur at the temperature- and current-dependent phase-slip rate $\Gamma(T, I)$ [27]. The first term corresponds to (deterministic) cooling as a result of conduction of heat from the central segment to the external bath via the two end segments, each of length $(L - l)/2$. The temperature-dependent cooling rate α is given by

$$\alpha(T, T_b) \equiv \frac{4}{l(L - l)C_v(T)}\frac{1}{T - T_b}\int_{T_b}^{T} dT' K_s(T'). \quad (3)$$

If T_i and T_f are temperatures before and after a phase slip then we can express the temperature "impulse" due to a phase slip, i.e., $T_f - T_i \equiv \eta(T_i, I) \equiv \tilde{\eta}(T_f, I)$, as function of either T_i or T_f (depending on the context) by using

$$Al\int_{T_i}^{T_f} C_v(T')dT' = \frac{hI}{2e}. \quad (4)$$

In writing these equation, in addition to restrictions on length scales, we assume that the time for a phase-slip ($\sim\tau_{GL}$) and the quasiparticle thermalization time τ_E are both smaller than the heat diffusion time $1/\alpha(T, T_b)$.

Let us now elucidate the physical and mathematical structure of Eq. (2). To begin with, we shall consider the continuous-heating limit, $\eta(T, I)\Gamma(T, I)$, for the source term, and express Eq. (2) as $dT/dt = -\partial U/dT$. In Fig. 2, we illustrate the form of the "potential" $U(T, T_b, I)$ for fixed T_b: there is a range of currents I for which U has two local minima, corresponding to the superconducting (at low-T) and resistive (at high-T) states, separated by a local maximum. In what follows, we focus on the stochastic variable $T(t)$; to ease the notation we do not display the dependences on I and T_b unless essential. To continue the analysis of the stochastic equation, we imagine turning off the (deterministic) cooling term. If we now start with an initial temperature T_0 then

$$T_0, \qquad T_0 + \eta(T_0), \qquad T_0 + \eta(T_0) + \eta[T_0 + \eta(T_0)], \ldots \quad (5)$$

defines the discrete sequence of values that phase slips

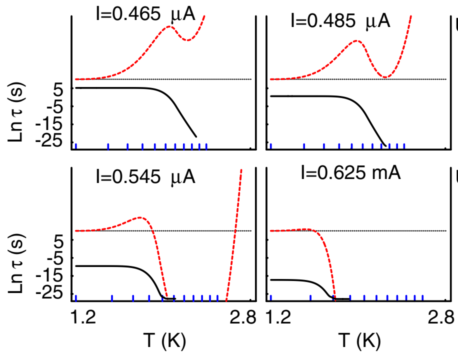

FIG. 2 (color online). Effective potential $U(T, T_b, I)$ (dashed line) and mean first-passage time τ as functions of the temperature T of the central segment for various bias currents I and for $T_b = 1.2$ K. The marks on the temperature axis indicate the temperatures that the central segment would have after $1, 2, \ldots, 10$ phase slips in the absence of cooling [as given by Eq. (5) for $T_0 = T_b$].

would cause T to jump to, as marked on the horizontal axes in Fig. 2 for $T_0 = T_b$. The probability per unit time, $\Gamma(T)$, to make a jump changes at each step, and so does the size $\eta(T)$ of the jump, owing to their explicit dependence on temperature. On the other hand, if we turn off the heating term then we would have a deterministic problem in which T would decay at a rate $\alpha(T)$, from its initial value $T_0 > T_b$ to the bath temperature T_b. It is the competition between the discrete heating and the continuous cooling that makes for a rather rich stochastic problem.

The master equation for $P(T, t)$, the probability for the central segment of the nanowire to have temperature T at time t reads

$$\partial_t P(T, t) = \partial_T[(T - T_b)\alpha(T)P(T, t)] - \Gamma(T)P(T, t) + \Gamma(T - \tilde{\eta}(T))P(T - \tilde{\eta}(T), t)(1 - \partial_T \tilde{\eta}(T)), \tag{6}$$

where the first (i.e., the transport) term corresponds to the effect of cooling, and the last two terms correspond to the effects of heating. Note that the term $(1 - \partial_T \tilde{\eta}(T))$ appears because of the dependence of the jump size on T, as given by $\tilde{\eta}(T)$.

The fundamental quantity of interest is the mean switching time $\tau_s(T_b, I)$, i.e., the mean time required for the wire to switch from being superconductive to resistive, assuming that the entire wire has temperature $T = T_b$ when the current I is turned on at time $t = 0$. The master equation (6) provides the starting point for generalizing the standard procedure for computing τ_s via the evaluation of the mean first-passage time [28], i.e., $\tau(T \to T^*)$, the time to go past a point $T = T^*$ for the first time having started from some $T < T^*$. τ can be shown to obey

$$-(T_b - T)\alpha(T)\partial_T \tau(T) + \Gamma(T)[\tau(T) - \tau(T + \eta(T))] = 1, \tag{7}$$

together with the conditions $\tau(T) = 0$ for $T > T^*$ and $d\tau(T)/dT = 0$ at $T = T_b$, which are appropriate for our problem. Some illustrative plots for $\tau(T \to T^*)$, obtained by numerically solving Eq. (7), are shown in Fig. 2, with the choice of T^* being somewhat larger than the location of the local maximum of U. As long as the high-T minimum is lower than the low-T one, and T^* is chosen to be appreciably past the intervening potential maximum (in order to eliminate the possibility of reversion to the superconducting state), we can make the identification: $\tau_s(T_b, I) \equiv \tau(T_b \to T^*, T_b, I)$. The number of tick marks [see the sequence (5)] between T_b and T^* is nothing but the number $N(T_b, I)$ of phase-slip events required to raise the temperature of the central segment from T_b to T^* in the absence of cooling. Accordingly, $N(T_b, I)$ also provides an estimate of the number of phase-slip events needed to overcome the potential barrier if the time span of these events is insufficient to allow significant cooling to occur. "Thermal runaway"—heating by rare sequences of closely spaced phase slips that overcome the potential barrier—constitutes the mechanism of superconductive-to-resistive switching within our model. As $N(T_b, I)$ becomes large, the total number of phase-slip events taking place before switching can happen, and, correspondingly, the value of $\tau_s(T_b, I)$, may indeed be quite large.

Our key findings are summarized in Fig. 3. There is a region of the $I - T_b$ plane for which the occurrence of *just one phase slip* (quantum or thermal) is sufficient to cause the nanowire to switch from the superconductive to the resistive state; in this case $\tau_s^{-1} = \Gamma$. Measurements in this range can thus provide a way of detecting and probing an individual phase-slip fluctuation. As, outside this range, several phase-slip events are required for switching, τ_s^{-1} deviates from Γ [see panel 3(c)]. A graphical representation of the contour lines for a few values of τ_s^{-1} and Γ, chosen in an experimentally accessible range, is provided in panel 3(a). While the spacing between the Γ contour lines decreases monotonically on lowering T_b, the spacing between the τ_s^{-1} lines can be seen to behave nonmonotonically.

The mean switching time τ_s in bistable current-biased systems can be either directly measured or extracted from the switching-current statistics generated via the repeated tracing of the I-V hysteresis loops [22,23]. For this reason, in Fig. 3(b) we have illustrated the behavior of this distribution of switching currents in superconducting nanowires based on the theory presented here. Upon raising T_b, one would naively expect the distribution to become broader for a model involving only thermally activated phase slips. Such broadening in the distribution width is indeed obtained up to a crossover temperature scale $T_b^{\rm cr}(r)$ (i.e., the temperature below which switching is induced by a single phase slip). However, on continuing to raise T_b, but now through temperatures above $T_b^{\rm cr}(r)$, the distribution width shows a seemingly anomalous decrease. This is a manifestation of the now-decreasing spacing between the τ_s contour lines. This striking behavior above $T_b^{\rm cr}$ may be

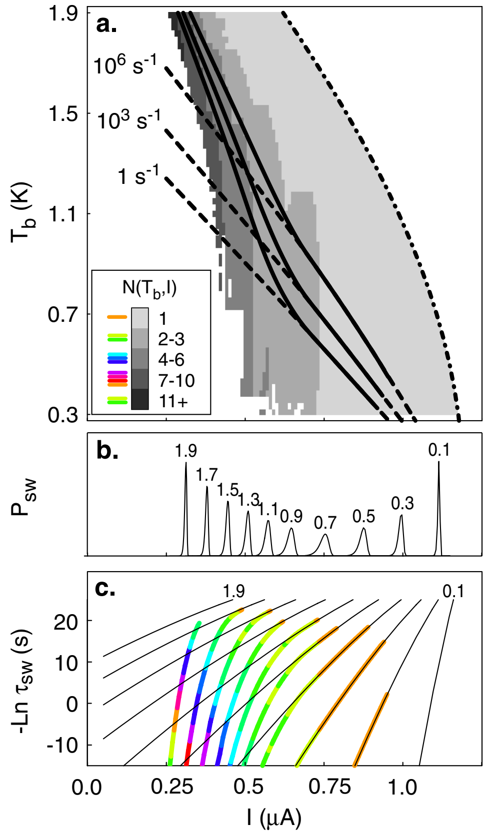

FIG. 3 (color online). Switching statistics. (a) Map showing $N(T_b, I)$ (see text) and the contour lines for the mean switching rates $\tau_s^{-1} = 1$, 10^3, 10^6 s^{-1} (solid) and the phase-slip rate Γ (dashed). The depairing critical current (dashed-dotted line) is plotted for reference. (b) Switching-current distributions P_{SW} obtained at various values of T_b and for sweep rate $r = 58$ μA/s. (c) The logarithms of τ_s^{-1} (colored lines) and of Γ (thinner black lines) as a function of I, obtained for the same set of T_b values as used in panel (a). The colors of the τ_s^{-1} plots correspond to different values of $N(T_b, I)$ [as indicated in the legend of panel (a)].

understood by the following reasoning: the larger the number of phase slips in the sequence inducing the superconductive-to-resistive thermal runaway, the smaller the stochasticity in the switching process and, hence, the sharper the distribution of switching currents.

The theory described is indeed found to furnish an explanation for the counter-intuitive increase of the distribution width obtained in a very recent experimental study of MoGe wires [23]. To be able to fit the data at low temperatures, we find that it is essential to incorporate QPS, and not just TAPS, into our model. At even lower temperatures, our model suggests that switching is caused by a single quantum phase slip. The switching current distribution thus promises to provide a powerful probe for accessing and thereby understanding the behavior of individual phase slips, thermal or quantum.

We acknowledge invaluable discussions with A. Bezryadin, M. Sahu, and T.-C. Wei, and, on the issue of numerical approaches, B. K. Clark. This work was supported by DOE grant DE-FG02-07ER46453 (D. P., P. M. G.) and NSF grant DMR 0605813 (N. S.).

[1] A. Bezryadin, C. N. Lau, and M. Tinkham, Nature (London) **404**, 971 (2000).
[2] D. S. Hopkins, D. Pekker, P. M. Goldbart, and A. Bezryadin, Science **308**, 1762 (2005).
[3] W. A. Little, Phys. Rev. **156**, 396 (1967).
[4] W. J. Skocpol and M. Tinkham, Rep. Prog. Phys. **38**, 1049 (1975).
[5] J. S. Langer and V. Ambegaokar, Phys. Rev. **164**, 498 (1967).
[6] D. E. McCumber and B. I. Halperin, Phys. Rev. B **1**, 1054 (1970).
[7] J. Bardeen, Rev. Mod. Phys. **34**, 667 (1962).
[8] A. V. Lopatin, N. Shah, and V. M. Vinokur, Phys. Rev. Lett. **94**, 037003 (2005).
[9] N. Shah and A. Lopatin, Phys. Rev. B **76**, 094511 (2007).
[10] A. D. Maestro, B. Rosenow, N. Shah, and S. Sachdev (to be published).
[11] A. D. Zaikin, D. S. Golubev, A. van Otterlo, and G. T. Zimányi, Phys. Rev. Lett. **78**, 1552 (1997).
[12] D. S. Golubev and A. D. Zaikin, Phys. Rev. B **64**, 014504 (2001).
[13] D. Meidan, Y. Oreg, and G. Refael, Phys. Rev. Lett. **98**, 187001 (2007).
[14] J. Romijn, T. M. Klapwijk, M. J. Renne, and J. E. Mooij, Phys. Rev. B **26**, 3648 (1982).
[15] A. J. Van Run, J. Romijn, and J. E. Mooij, Jpn. J. Appl. Phys. **26**, 1765 (1987).
[16] N. Giordano, Phys. Rev. B **41**, 6350 (1990).
[17] A. Rogachev, A. T. Bollinger, and A. Bezryadin, Phys. Rev. Lett. **94**, 017004 (2005).
[18] F. Altomare, A. M. Chang, M. R. Melloch, Y. Hong, and C. W. Tu, Phys. Rev. Lett. **97**, 017001 (2006).
[19] M. Zgirski, K.-P. Riikonen, V. Touboltsev, and K. Yu. Arutyunov, Phys. Rev. B **77**, 054508 (2008).
[20] F. Mattioli, R. Leoni, A. Gaggero, M. G. Castellano, P. Carelli, F. Marsili, and A. Fiore, J. Appl. Phys. **101**, 054302 (2007).
[21] J. E. Mooij and Y. V. Nazarov, Nature Phys. **2**, 169 (2006).
[22] T. A. Fulton and L. N. Dunkleberger, Phys. Rev. B **9**, 4760 (1974).
[23] M. Sahu, M.-H. Bae, A. Rogachev, D. Pekker, T.-C. Wei, N. Shah, P. M. Goldbart, and A. Bezryadin, arXiv:0804.2251v1.
[24] M. Tinkham, J. U. Free, C. N. Lau, and N. Markovic, Phys. Rev. B **68**, 134515 (2003).
[25] M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1996).
[26] J. Bardeen, G. Rickayzen, and L. Tewordt, Phys. Rev. **113**, 982 (1959).
[27] In this Letter, for the purpose of sample calculations, we shall assume that $\Gamma(T)$ is given by the TAPS rate [6]. QPS may be easily incorporated by including them in $\Gamma(T)$.
[28] N. G. Van Kampen, *Stochastic Processes in Physics and Chemistry* (North-Holland, Amsterdam, 1992).